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Reflection Lines

The description of reflection lines in the book will yield reflection lines that correspond to finite - length light lines. If the light lines are meant to be of infinite length, then the following will work.

Let $\mathbf{x}$ be a point on a surface, and let $\mathbf{n}$ be its normal. Let a light line source be defined by a point $\mathbf{p}$ and a vector $\mathbf{v}$. Denote by $\mathbf{P}$ the plane through $\mathbf{x}$ with normal vector $\mathbf{v}$. We compute two points: $\mathbf{q}$, the projection of the point $\mathbf{x}+\mathbf{n}$ into $\mathbf{P}$ as well as $\mathbf{r}$, the intersection of $\mathbf{L}$ with $\mathbf{P}$. See Figure 1.

We write $\mathbf{r}$ as $\mathbf{r} = \mathbf{p} + t\mathbf{v}$ and get the condition

$$(\mathbf{p} + t\mathbf{v} - \mathbf{x})\mathbf{v} = 0,$$

thus obtaining

$$t = \frac{\mathbf{x}\mathbf{v} - \mathbf{p}\mathbf{v}}{\mathbf{v}\mathbf{v}}.$$

We write $\mathbf{q}$ as $\mathbf{q} = \mathbf{x} + \mathbf{n} + s\mathbf{v}$ and obtain

$$(\mathbf{x} + \mathbf{n} + s\mathbf{v} - \mathbf{x})\mathbf{v} = 0,$$

thus obtaining (note two $\mathbf{x}$'s cancel):

$$s = \frac{-\mathbf{n}\mathbf{v}}{\mathbf{v}\mathbf{v}}.$$

Now we use the angle α formed by the vectors $\mathbf{q} - \mathbf{x}$ and $\mathbf{r} - \mathbf{x}$ to determine if the point $\mathbf{x}$ reflects light or not.

Figure 1: The reflection line geometry.