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Practical Linear Algebra – A Geometry Toolbox

Gerald Farin & Dianne Hansford

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Linear Algebra Glossary

In this Glossary, we give brief definitions of the major concepts in the book. We try to avoid equations here, so that we give a slightly different perspective compared to what you find in the text.

Affine map A map which leaves geometric (i.e., linear) relationships between points unchanged. For instance, midpoints are mapped to midpoints. In a given coordinate system, an affine map is described by a transformation matrix and a translation vector.

Affine space A set of points with the property that any barycentric combination of two points is again in the space.

Barycentric combination A weighted average of points where the sum of the weights equals one.

Barycentric coordinates When a point is expressed as a barycentric combination of the three vertices of a triangle, the coefficients in that combination are called barycentric coordinates.

Basis For a linear space of dimension n, any set of n linearly independent vectors is a basis, meaning that every vector in the space may be uniquely expressed as a linear combination of these n basis vectors.

Basis transformation The linear map taking one set of n basis vectors to another set of n basis vectors.

Centroid The center of mass, or the average of a set of points, with all weights being equal (and summing to one).

Collinear A set of points is called collinear if they all lie on the same straight line.

Condition number A function measuring how sensitive a map is to changes in its input. If a small changes in the input cause large changes in the output, then the condition number is large.

Conic section The intersection curve of a double cone with a plane. A nondegenerate conic section is either an ellipse, a parabola, or a hyperbola.

Convex A point set is convex if the straight line segment through any of its points is completely contained inside the set. Example: all points on and inside of a sphere form a convex set; all points on and inside of an hourglass do not.

Coordinates A vector in an $n-$dimensional linear space may be uniquely written as a linear combination of a set of basis vectors. The coefficients in that combination are the vector's coordinates with respect to that basis.

Coplanar A set of points is coplanar if all points lie on the same plane.

Cramer's Rule A method for solving a linear system explicitly using ratios of determinants.

Cross product The cross product of two 3D vectors results in a third vector which is perpendicular to them.

Curve The locus of a moving point.

Determinant A linear map takes a geometric object to another geometric object. The ratio of their volumes is the map's determinant.

Dimension The number of linearly independent vectors needed to span a linear space.

Dot product assigning a value to two vectors which, in the case of unit vectors, is equal to the cosine of the angle between the vectors.

Dual space Consider all linear maps from a linear space into the 1D linear space of scalars. All these maps form a linear space themselves, the dual space of the original space.

Eigenvalue If a linear map happens to take some vector to itself, multiplied by some constant, then that constant is an eigenvalue of the map.

Eigenvector A vector whose direction is unchanged by a linear map.

Ellipse A bounded conic section. When written in implicit form, its 2x2 matrix has two positive eigenvalues.

Gauss elimination The process of transforming a linear system into an equivalent linear system whose coefficient matrix is of upper triangular form.

Gauss-Seidel iteration Solving a linear system by successively improving an initial guess for the solution. Similar to Gauss-Jacobi iteration.

Homogeneous coordinates Points in 2D affine space may be viewed as projections of points in 3D affine space, all being multiples of each other. The coordinates of any of these points are the homogeneous coordinates of the given point.

Homogeneous linear system A linear system whose right hand side consists of zeroes only.

Hyperbola An unbounded conic section with two branches. When written in implicit form, its 2x2 matrix has a positive and a negative eigenvalue.

Idempotent A map is idempotent if repeated applications of the map yield the same result as only one application. Example: projections.

Identity matrix A square matrix with entries 1 on the diagonal and entries 0 elsewhere. This matrix maps every object to itself.

Image The result of a map.

Incenter There is exactly one circle inside a triangle which has its three edges as tangents. The center of this circle is the triangle's incenter.

Inner product Given two elements of a linear space, their inner product is a scalar. If the two vectors are orthogonal, then their inner product is zero.

Inverse matrix A matrix maps an object to another object. The inverse matrix undoes this map.

Kernel The set of vectors being mapped to the zero vector by a linear map. Also called the null space.

Least squares A method for finding the best approximate solution to an overdetermined problem.

Line Given two points in affine space, the set of all barycentric combinations is a line.

Line segment As above, but with all coefficients of the barycentric combinations being nonnegative.

Linear combination A weighted sum of vectors.

Linear independence A set of vectors is called linearly independent if none of its elements may be written as a linear combination of the remaining ones.

Linear interpolation A weighted average of two points, where the weights sum to one and are linear functions of a parameter.

Linear map A map of a linear space to another linear space such that linear relationships between points are not changed by the map. In a given coordinate system, a linear map is described by a matrix.

Linear space A set (whose elements are called vectors) with the property that any linear combination of any two vectors is also in the set.

Linear system If we attempt to write a given vector as a linear combination (with unknown coefficients) of a set of vectors, then the resulting set of equations is called a linear system.

Length The distance between two points forming a line segment.

Local coordinates A specific coordinate system used to define a geometric object. This object may then be placed in a global coordinate system.

Map The process of changing objects. Example: rotating and scaling an object. The object being mapped is called the preimage, the result of the map is called the image.

Matrix The coordinates of a linear map, written in a rectangular array of scalars.

Nonlinear map A map which does not preserve linear relationships. Example: a perspective map.

Norm A function which assigns a length to a vector.

Null space The set of vectors which are mapped to the zero vector by a linear map is called that map's null space. Also called the kernel.

Orthogonality Two vectors are orthogonal if their dot product vanishes.

Parabola An unbounded conic section with one branch. When written in implicit form, its 2x2 matrix has one zero eigenvalue.

Parallel Two lines or two planes are parallel if they have no point in common. Two vectors are called parallel if they are multiples of each other.

Plane Given three points in affine space, the set of all barycentric combinations is a plane.

Point A location, i.e., an element of an affine space.

Point Cloud A set of 3D points without any additional structure.

Polygon The set of edges formed by connecting a set of points.

Projection A linear map which reduces the dimension of an object.

Range The set of possible images resulting from a given map.

Rank Any set of vectors contains a maximal number of linearly independent vectors. If the vectors are arranged in a matrix, then that number is the rank of the matrix.

Ratio A measure of how three collinear points are distributed. If one is the midpoint of the other two, the ratio is 1.

Rigid body motion An affine map which leaves distances and angles unchanged.

Singular matrix A matrix describing a linear map which is a projection, meaning that it reduces the dimension of an object.

Span For a given set of vectors, its span is the set (space) of all vectors which can be obtained as linear combinations of these vectors.

Subspace A set of linearly independent vectors defines a linear space. Any subset of these vectors defines a subspace of that linear space.

Triangulation Also called triangle mesh: a set of 2D or 3D points which is faceted into nonoverlapping triangles.

Unit vector A vector whose length is 1.

Vector An element of a linear space – or equivalently, the difference of two points in an affine space.

Zero vector A vector of zero length. Every linear space contains a zero vector.