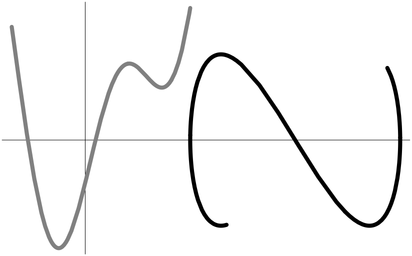
# Calculus Background

**Gerald Farin, Dianne Hansford**
Arizona State University

August 24, 2008

# Functions

$$x \rightarrow f(x)$$

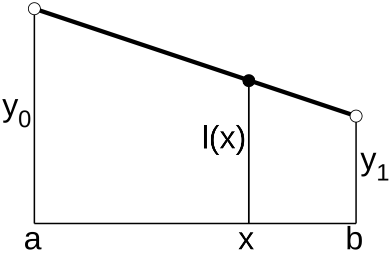


# Attributes

- Continuity
- Monotonicity
- Convexity
- Symmetry
- Boundedness

# Linear Functions

$$l(x) = \frac{b-x}{b-a}y_0 + \frac{x-a}{b-a}y_1$$



# Combinations

Linear combination

$$h(x) = \alpha f(x) + \beta g(x)$$

Product

$$h(x) = f(x) \cdot g(x)$$

Composition

$$h(x) = f(g(x))$$

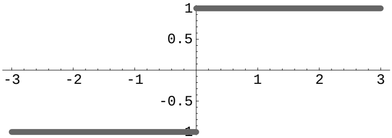
Example: $f(x) = x^2$, $g(x) = x + 1$, $h(x) = (x + 1)^2$

# Limits

$$f(x) = \frac{|x|}{x}$$

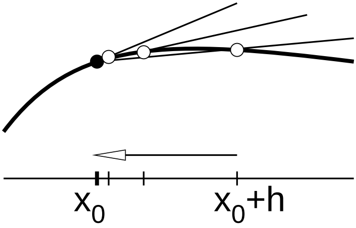
$$\lim_{x \to 0+} f(x) = 1$$

$$\lim_{x \to 0-} f(x) = -1$$



# Example: Derivatives

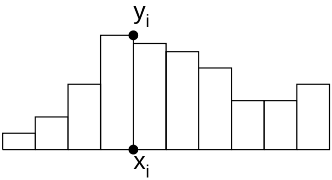
$$\lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$



# Integrals I

Area:

$$A = \sum_{i=0}^{N-1} \Delta x_i y_{i+1}$$



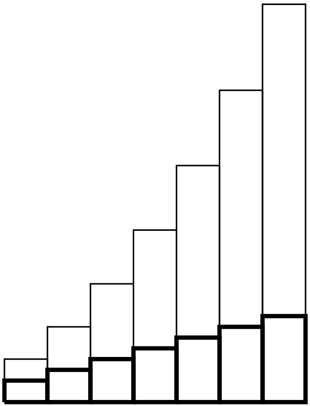
# Integrals II

Define:

$$F(x_0) = c,$$

$$F(x_{i+1}) = F(x_i) + \Delta x_i f(x_{i+1})$$

$$A = F(x_N) - F(x_0)$$



# Integral as a Limit

$N \to \infty$:

$$A = \int_a^b f(x)\mathrm{d}x.$$

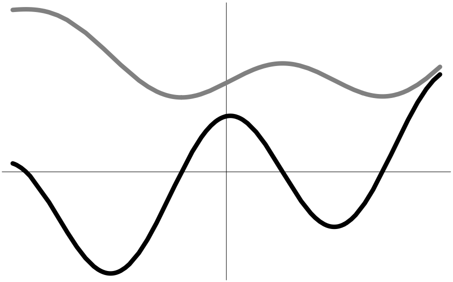
# Derivatives

Rewrite definition of $F$:

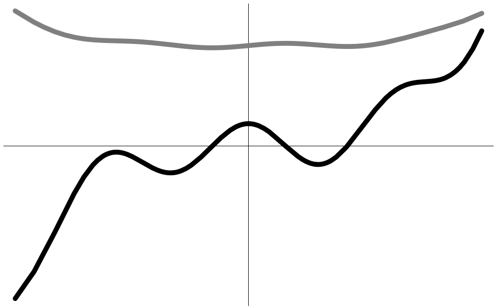
$$f(x_{i+1}) = \frac{\Delta F(x_i)}{\Delta x_i}$$

Limit:

$$f(x) = \frac{\mathrm{d}F(x)}{\mathrm{d}x}$$



# Derivatives Roughen



# Function Spaces

]
Cubics:

$$p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$$p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \ \text{ and } \ q(x) = b_0 + b_1 x + b_2 x^2 + b_3 x^3,$$

$$\alpha p(x) + \beta q(x) = (a_0 + b_0) + (a_1 + b_1)x + (a_2 + b_2)x^2 + (a_3 + b_3)x^3,$$

Cubics form **linear space** $\mathbb{P}^3$.

# $\mathbb{P}^3$ Dimension

Basis:

$$1, x, x^2, x^3$$

$$\dim \mathbb{P}^3 = 3$$