Coordinate Systems

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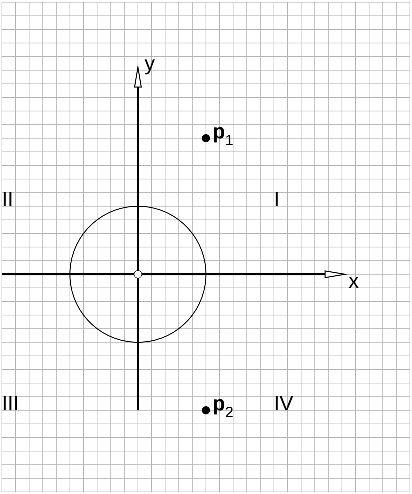
2D Cartesian

circle = unit circle

$\mathbf{p}_1$ has (x, y)-coordinates $(1, 2)$

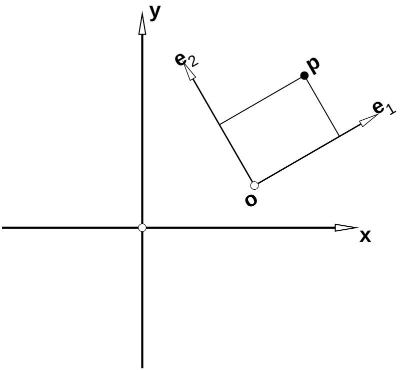
$\mathbf{p}_2$ has (x, y)-coordinates $(1, -2)$



2D Cartesian

$$\mathbf{o} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\mathbf{p} = \mathbf{o} + p_1\mathbf{e}_1 + p_2\mathbf{e}_2.$$

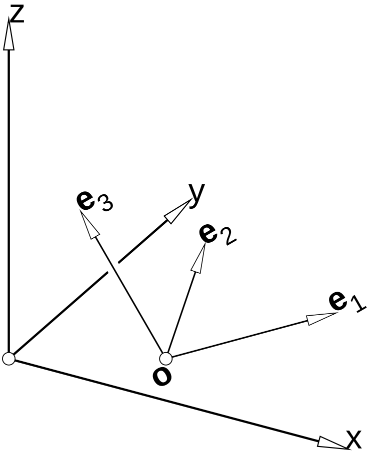


3D Cartesian

$$\mathbf{o} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

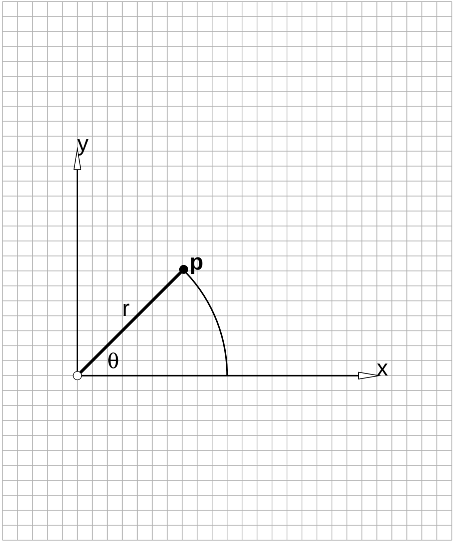
$$\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{p} = \mathbf{o} + p_1\mathbf{e}_1 + p_2\mathbf{e}_2 + p_3\mathbf{e}_3.$$



Polar

$$r = \|\mathbf{p}\| \quad \Theta = \text{modulus} \;\; \mathbf{p}$$

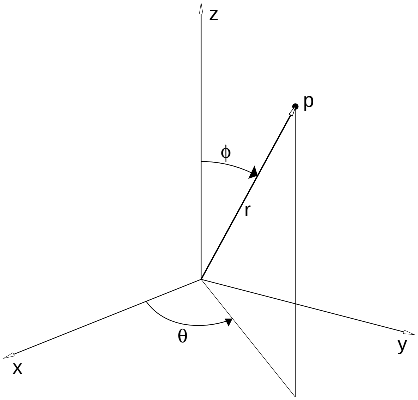


Spherical

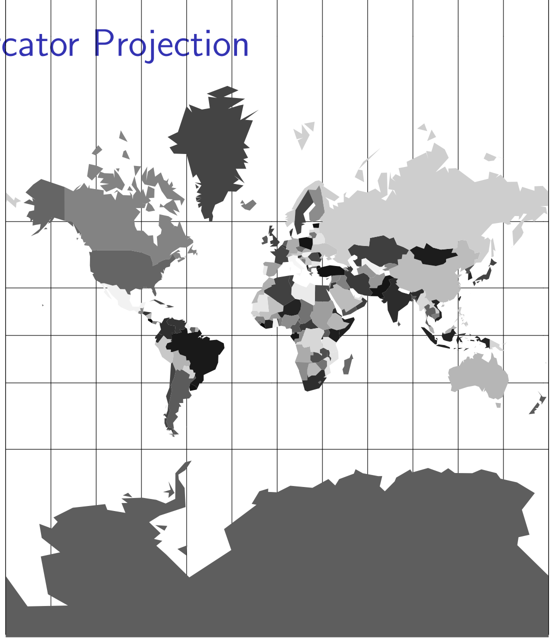
$$r = \|\mathbf{p}\|$$

$$\Theta = \text{rotation around } z$$

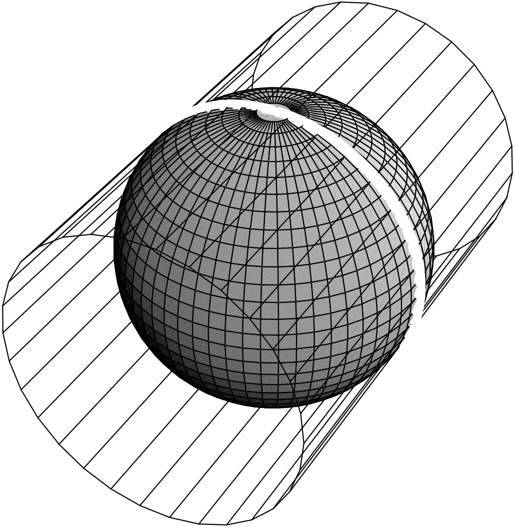
$$\Phi = \text{rotation around } x$$



Mercator Projection

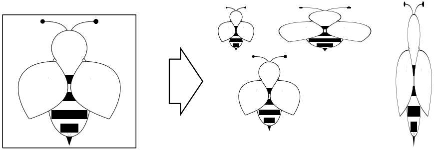


UTM Coordinates

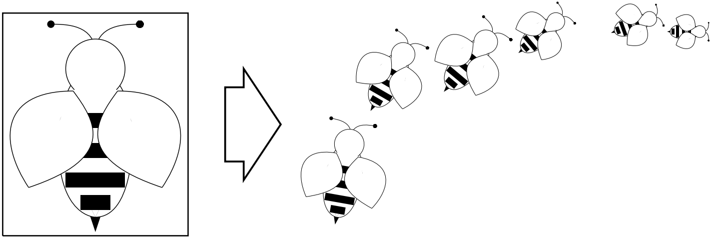


Local vs Global

$$x_1 = (1 - u_1)\mathrm{min}_1 + u_1\mathrm{max}_1$$
$$x_2 = (1 - u_2)\mathrm{min}_2 + u_2\mathrm{max}_2$$



Local vs Global



Homogeneous Corrdinates

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \longrightarrow \begin{bmatrix} x/z \\ y/z \\ 1 \end{bmatrix} = \mathbf{y}$$