Roots

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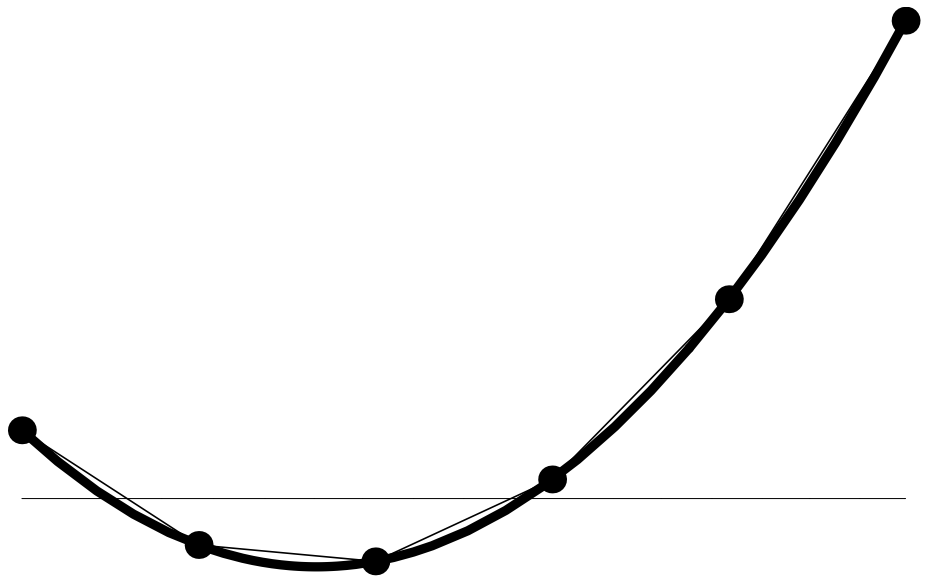
Piecewise Linear

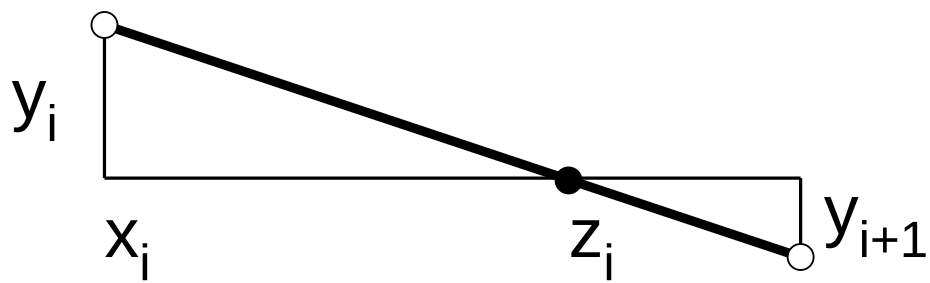
Replace function by pw linear approximation.



Piecewise Linear

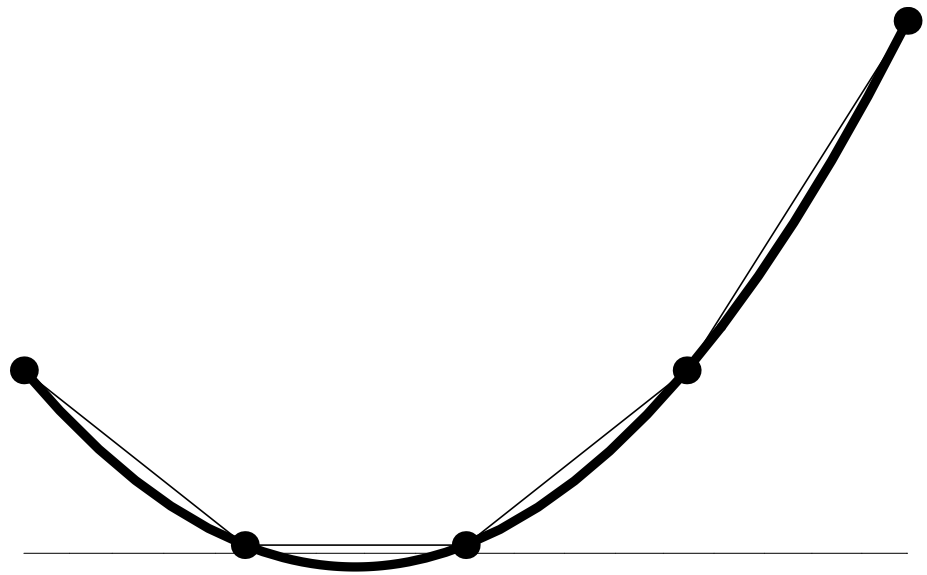
find zero z_i from:

$$\frac{-y_i}{z_i - x_i} = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}$$



Piecewise Linear

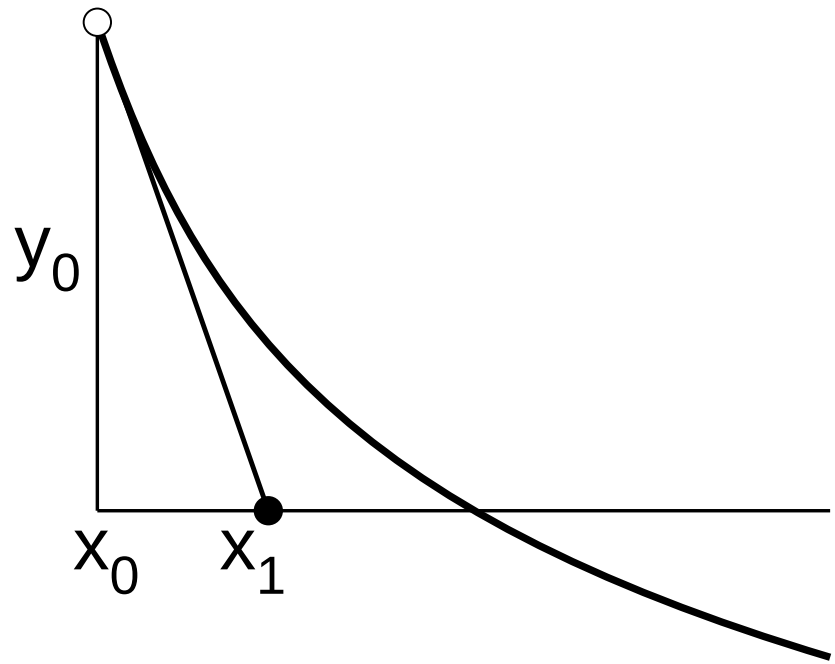
zeroes may be missed!



Newton-Raphson

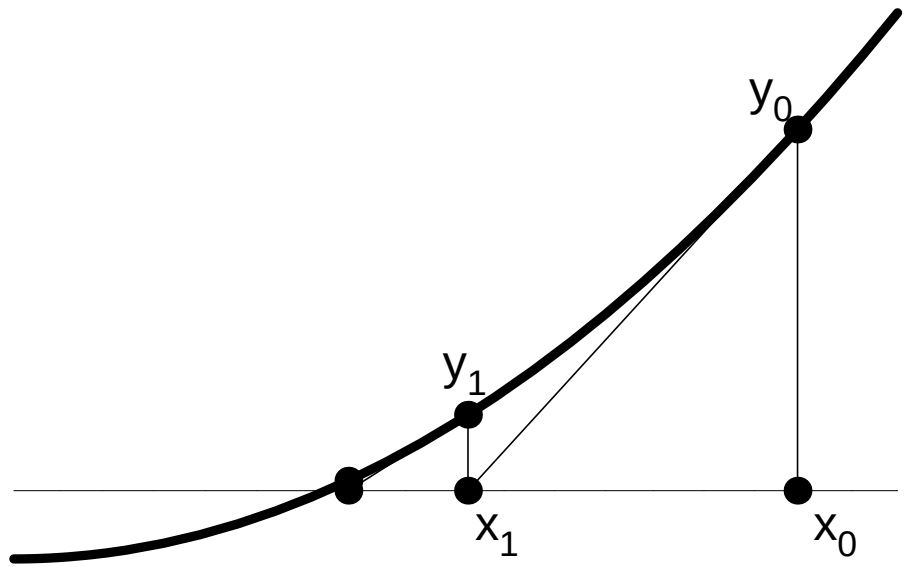
$$y_0' = \frac{y_0}{x_0 - x_1}$$

$$x_1 = x_0 - \frac{y_0}{y_0'}$$



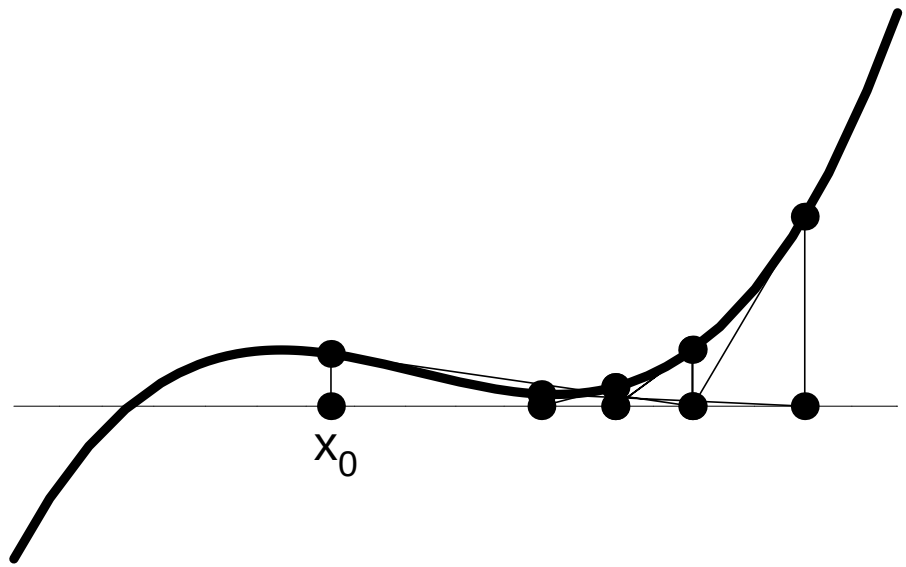
Newton-Raphson

...Repeat until done



Newton-Raphson

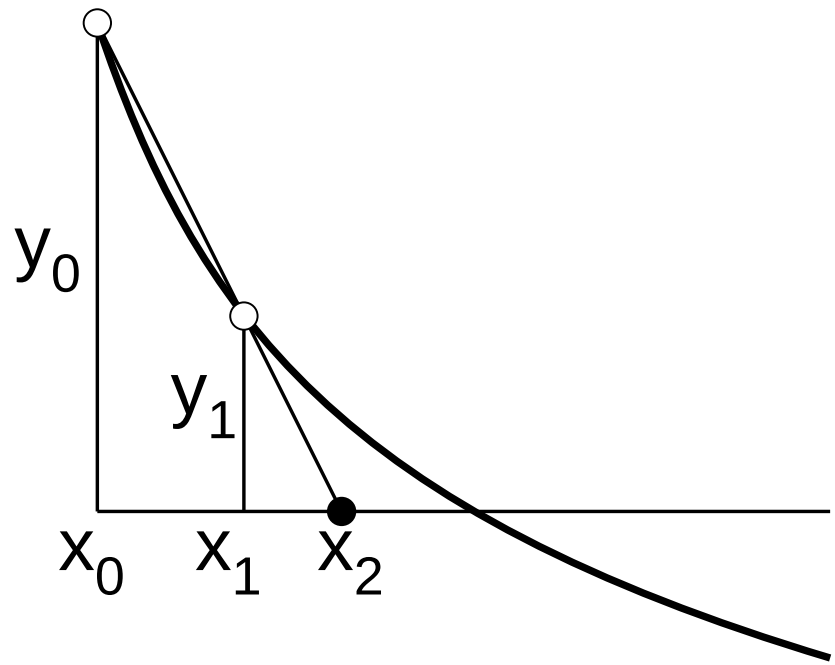
May fail!



Secant Method

$$\frac{y_0 - y_1}{x_0 - x_1} = \frac{y_0}{x_0 - x_2}$$

$$x_2 = x_0 - y_0 \frac{x_0 - x_1}{y_0 - y_1}$$



Case Study: Square Root

$$ax^2 + bx + c = 0$$

exact solution

$$x = \frac{1}{2a}[-b \pm \sqrt{b^2 - 4ac}]$$

No *exact* way to compute $\sqrt{\ }$.

Square Root

Problem:

$$x = \sqrt{a}$$

$$f(x) = x^2 - a.$$

$f'(x) = 2x$, thus

$$x_{i+1} = x_i - \frac{x_i^2 - a}{2x_i}.$$

$a = 2$ and $x_0 = 1$:

1.
1.50000
1.41667
1.41422
1.41421
1.41421

Example: Wilkinson Polynomials

$$w_n(t) = (t-1) \cdot (t-2) \cdot \ldots \cdot (t-n)$$

For $n > 23$, Mathematica fails to find roots.