

Facets

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Triangles

Triangle: 2D or 3D, defined by vertices $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$

Centroid:

$$\mathbf{c} = \frac{1}{3}(\mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3)$$

Normal:

$$\mathbf{n} = \frac{(\mathbf{v}_2 - \mathbf{v}_1) \wedge (\mathbf{v}_3 - \mathbf{v}_1)}{\|(\mathbf{v}_2 - \mathbf{v}_1) \wedge (\mathbf{v}_3 - \mathbf{v}_1)\|}$$

Area (2D or 3D):

$$\mathcal{A} = \sqrt{(\mathbf{v}_2 - \mathbf{v}_1)^2(\mathbf{v}_3 - \mathbf{v}_1)^2 - [(\mathbf{v}_2 - \mathbf{v}_1)(\mathbf{v}_3 - \mathbf{v}_1)]^2}$$

Barycentric Coordinates

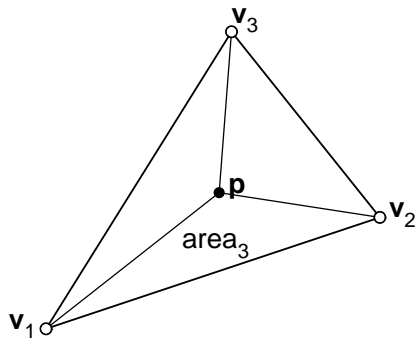
$$\mathbf{p} = u_1\mathbf{v}_1 + u_2\mathbf{v}_2 + u_3\mathbf{v}_3$$

$$u_1 = \text{area}[\mathbf{p}, \mathbf{v}_2, \mathbf{v}_3] / \mathcal{A}$$

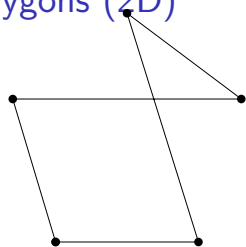
$$u_2 = \text{area}[\mathbf{p}, \mathbf{v}_3, \mathbf{v}_1] / \mathcal{A}$$

$$u_3 = \text{area}[\mathbf{p}, \mathbf{v}_1, \mathbf{v}_2] / \mathcal{A}$$

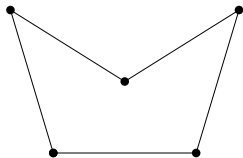
$$u_1 + u_2 + u_3 = 1$$



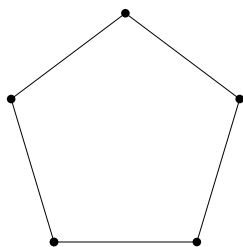
Polygons (2D)



general



simple



convex

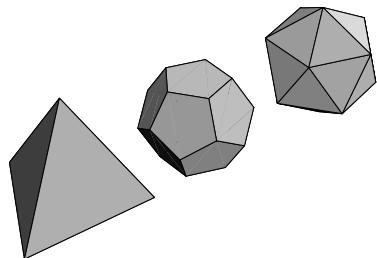
Area:

$$A = \frac{1}{2}[x_1y_2 - y_1x_2 + \dots + x_py_1 - y_px_1].$$

vertices: $\mathbf{x}_1, \dots, \mathbf{x}_p$

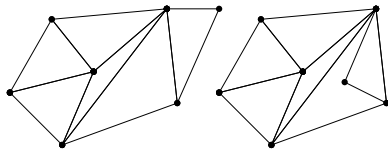
Polyhedra (3D)

Polyhedron: bounded by planar convex polygons



Triangle Meshes

Two triangles: either a vertex or an edge in common



Data Structure

Point list:

v1: x1, y1, z1

v2: x2, y2, z2

...

vN: xN, yN, zN

Triangle list:

T1: v11, v12, v13

T2: v21, v22, v23

...

TK: vK1, vK2, vK3

Neighbor list:

N1: n11, n12, n13

N2: n21, n22, n23

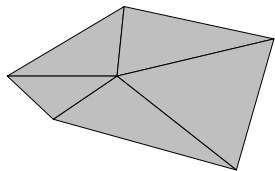
...

NK: nK1, nK2, nK3

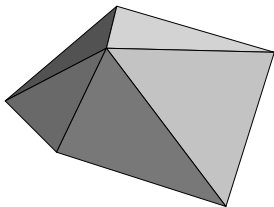
Case Study: 3D archiving

<http://www-graphics.stanford.edu/projects/mich/>

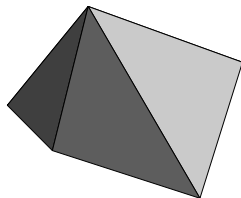
Mesh Shape



low curv.



med. curv.



hi curv.

Curvature

$$K_i = \frac{2\pi - S_i}{3|\mathbf{v}_i^*|}$$

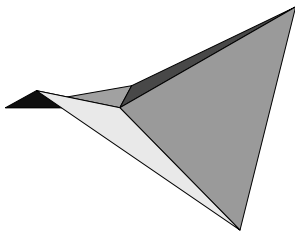
S_i = sum of angles at vertex \mathbf{v}_i

$|\mathbf{v}_i^*|$ = sum of triangles areas at \mathbf{v}_i

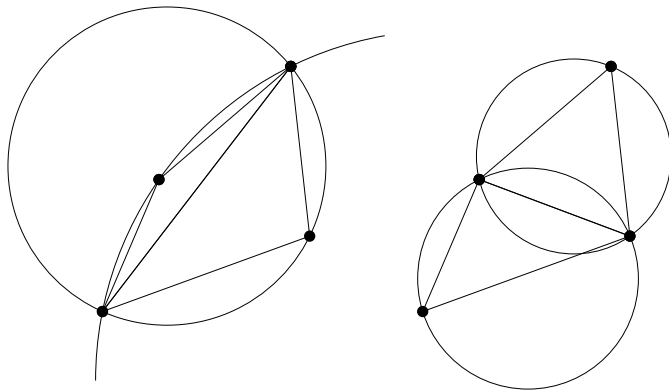
Curvature

Negative curvature:

$$S_i > 2\pi$$

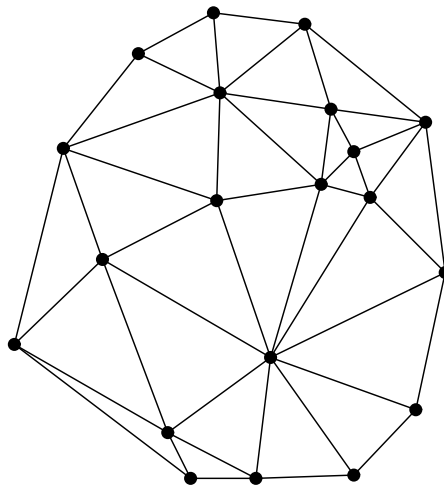
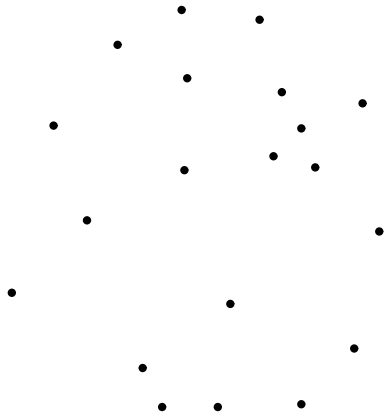


Delaunay Meshes (2D)

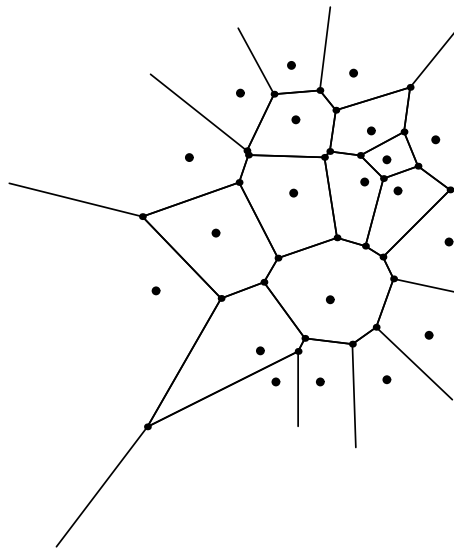
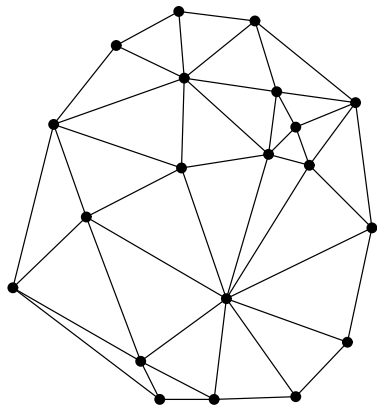


Delaunay \approx empty circumcircle
left: not Delaunay; right: Delaunay

Delaunay Meshes (2D)



Voronoi Diagrams (2D)



Quad Meshes (3D)

