

ASU ID:

Name:

CPI 200

October 19, 2009

G. Farin

**Midterm**

Closed Book, closed notes

Let  $\mathbb{R}, \mathbb{Q}, \mathbb{N}$  define the reals, rationals, and natural numbers resp.

1. Is  $\mathbb{N} \subset \mathbb{R} \cup \mathbb{N}$ ? Explain.

A: We know  $\mathbb{N} \subset \mathbb{R}$ . Also,  $\mathbb{N} \cup \mathbb{R} = \mathbb{R}$ . Hence the answer is yes.

2. Let  $A$  and  $B$  be sets such that  $A \subset B$ . What is  $A \cup B \cup \Phi$  where  $\Phi$  is the empty set?

A:  $B$  contains all the sets in  $A \cup B \cup \Phi$ , hence the answer is  $B$ .

4. Give the truth table for the logical expression

$$p \vee (p \wedge q)$$

A:

$p$	$q$	$p \wedge q$	$p \vee (p \wedge q)$
$F$	$F$	$F$	$F$
$F$	$T$	$F$	$F$
$T$	$F$	$F$	$T$
$T$	$T$	$T$	$T$

5. Which set is defined by the the following recursion:

$$p(n) = \begin{cases} 1 & \text{if } n = 0, \\ 2p(n-1) + 1 & \text{else.} \end{cases}$$

It will be sufficient if you give the 4 first elements.

A: 1,3,7,15,...

6. Find a recursion for the set  $\{1, -1, 1, -1, \dots\}$ .

A:

$$p(n) = \begin{cases} 1 & \text{if } n = 1, \\ -p(n-1) & \text{else.} \end{cases}$$

7. Find

$$\lim_{n \rightarrow \infty} \frac{3n^2 + 1}{6n^3 - 1}$$

and explain your answer.

A: The growth of the denominator exceeds that of the numerator, hence the limit is 0.

8. Is the function

$$y = 1 + x + x^2$$

even, odd, or neither? Explain.

A: neither, since it contains both even and odd powers of  $x$ .

9. As  $x \rightarrow \infty$ , what function does

$$y = x^2 + \frac{1}{x} \sin(x)$$

A: the limit of  $\frac{1}{x} \sin(x)$  is zero since the sine function has no growth. Thus the limit function is  $y = x^2$ .

10. Is the function

$$y = \frac{|x|}{x}$$

continuous? Explain.

A: This function may also be written as

$$y = \begin{cases} -1 & \text{if } x < 0, \\ 1 & \text{if } x > 0. \end{cases}$$

Thus it has a jump discontinuity at  $x = 0$  and is not a continuous function.