

# Introduction to Set Theory

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# Sets

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- # *Definition.* A *Set* is any well defined collection of “objects.”
  - # *Definition.* The *elements* of a set are the objects in a set.
  - # *Notation.* Usually we denote sets with upper-case letters, elements with lower-case letters. The following notation is used to show set membership
  - #  $x \in A$  means that  $x$  is a member of the set  $A$
  - #  $x \notin A$  means that  $x$  is not a member of the set  $A$ .
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# Ways of Describing Sets

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# List the elements

$$A = \{1, 2, 3, 4, 5, 6\}$$

# Give a verbal description

# “A is the set of all integers from 1 to 6, inclusive”

# Give a mathematical inclusion rule

$$A = \{\text{Integers } x \mid 1 \leq x \leq 6\}$$

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# Some Special Sets

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- # The Null Set or Empty Set. This is a set with no elements, often symbolized by

$\emptyset$

- # The Universal Set. This is the set of all elements currently under consideration, and is often symbolized by

$\Omega$

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# Membership Relationships

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# *Definition.* Subset.

$A \subseteq B$  “A is a subset of B”

We say “A is a subset of B” if  $x \in A \Rightarrow x \in B$ , i.e., all the members of A are also members of B. The notation for subset is very similar to the notation for “less than or equal to,” and means, in terms of the sets, “included in or equal to.”

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# Membership Relationships

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# *Definition.* Proper Subset.

$A \subset B$      “A is a proper subset of B”

We say “A is a proper subset of B” if all the members of A are also members of B, but in addition there exists at least one element  $c$  such that  $c \in B$  but  $c \notin A$ . The notation for subset is very similar to the notation for “less than,” and means, in terms of the sets, “included in but not equal to.”

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# Combining Sets – Set Union

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#  $A \cup B$

- # “ $A$  union  $B$ ” is the set of all elements that are in  $A$ , or  $B$ , or both.
  - # This is similar to the logical “or” operator.
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# Combining Sets – Set Intersection

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#  $A \cap B$

- # “ $A$  intersect  $B$ ” is the set of all elements that are in *both*  $A$  and  $B$ .
  - # This is similar to the logical “and”
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# Set Complement

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#  $\overline{A}$

- # “ $A$  complement,” or “not  $A$ ” is the set of all elements not in  $A$ .
- # The complement operator is similar to the logical not, and is reflexive, that is,

$$\overline{\overline{A}} = A$$

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# Set Difference

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$$A - B$$

- # The set difference “A minus B” is the set of elements that are in A, with those that are in B subtracted out. Another way of putting it is, it is the set of elements that are in A, *and* not in B, so  $A - B = A \cap \bar{B}$
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# Examples

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$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

$$A = \{1, 2, 3\} \quad B = \{3, 4, 5, 6\}$$

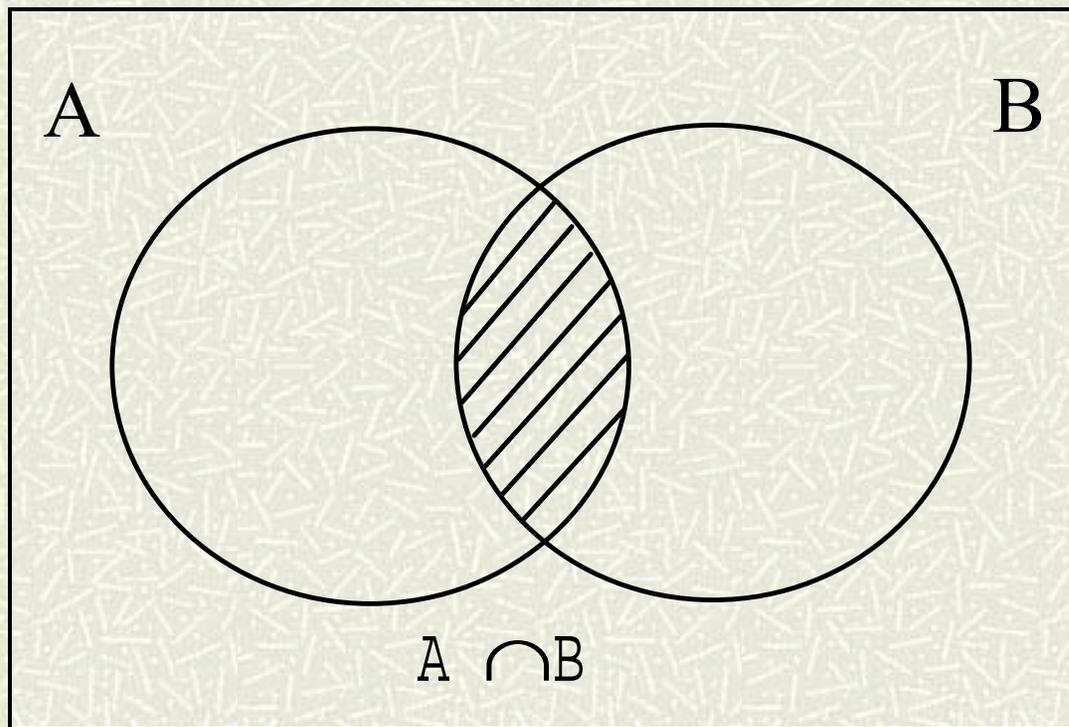
$$A \cap B = \{3\} \quad A \cup B = \{1, 2, 3, 4, 5, 6\}$$

$$B - A = \{4, 5, 6\} \quad \bar{B} = \{1, 2\}$$

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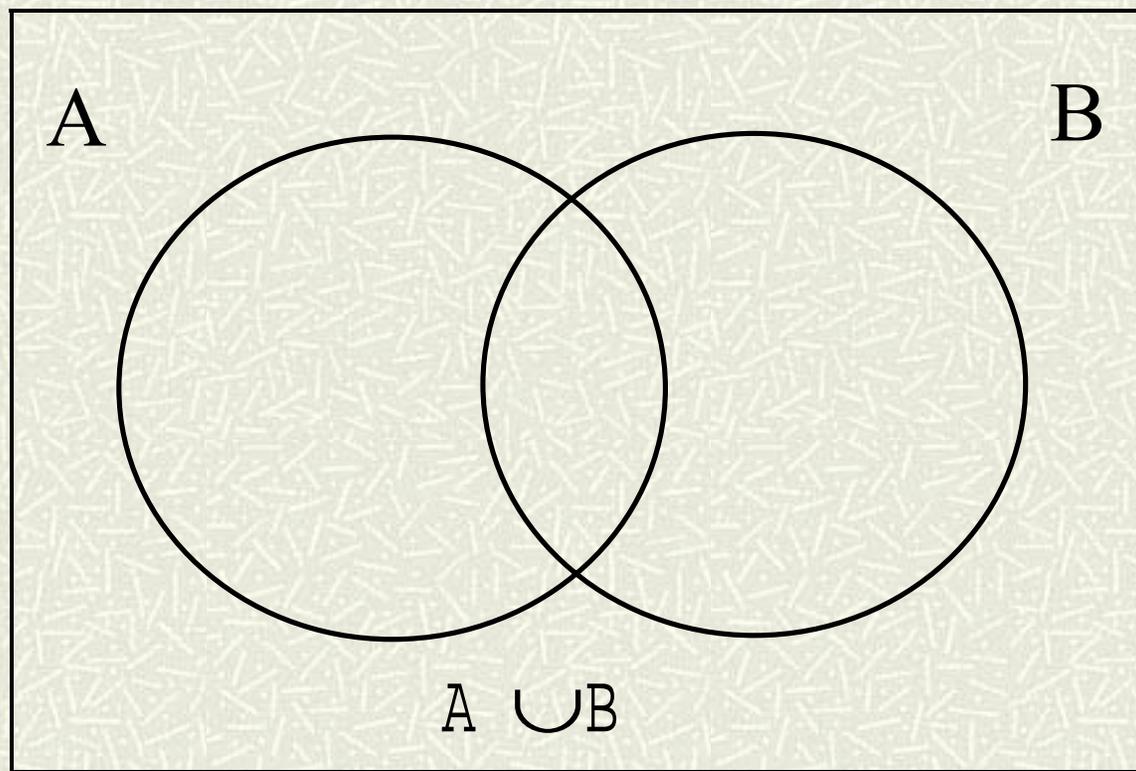
# Venn Diagrams

# **Venn Diagrams** use topological areas to stand for sets. I've done this one for you.



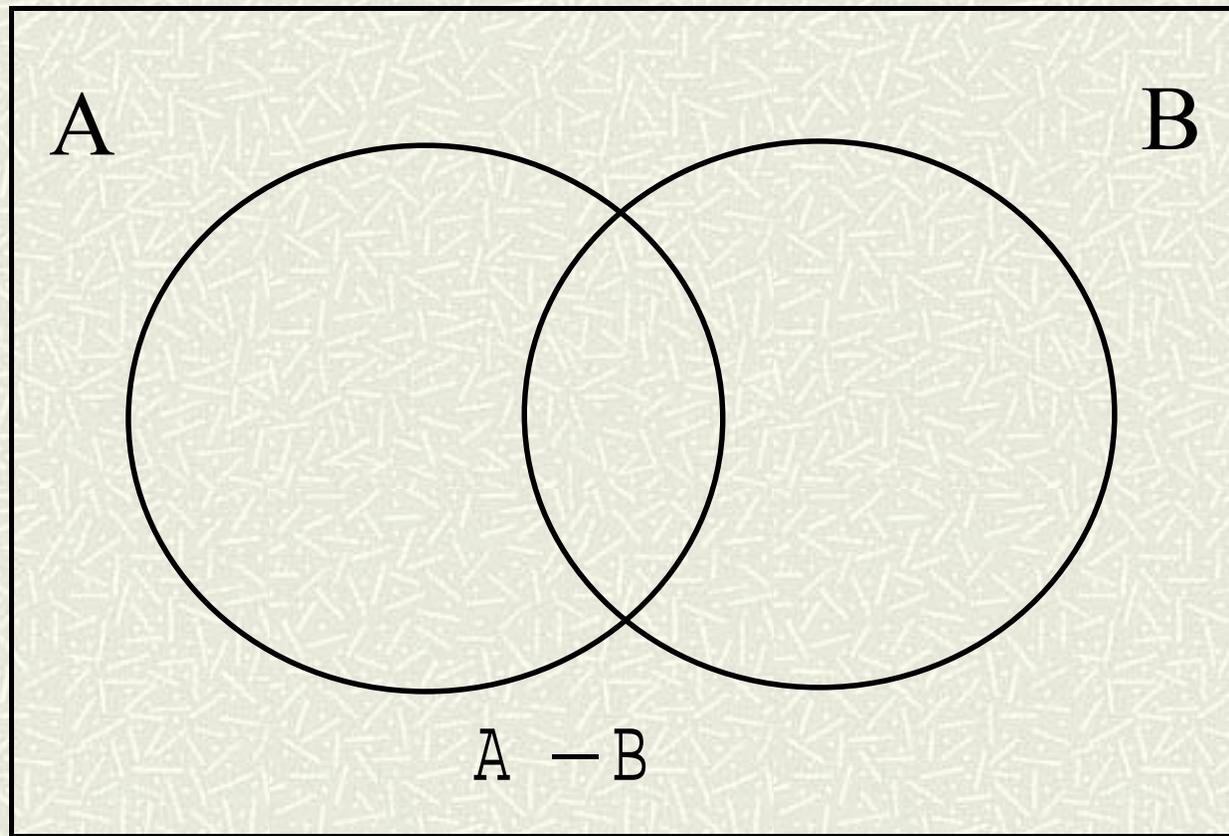
# Venn Diagrams

# Try this one!



# Venn Diagrams

# Here is another one



# Mutually Exclusive and Exhaustive Sets

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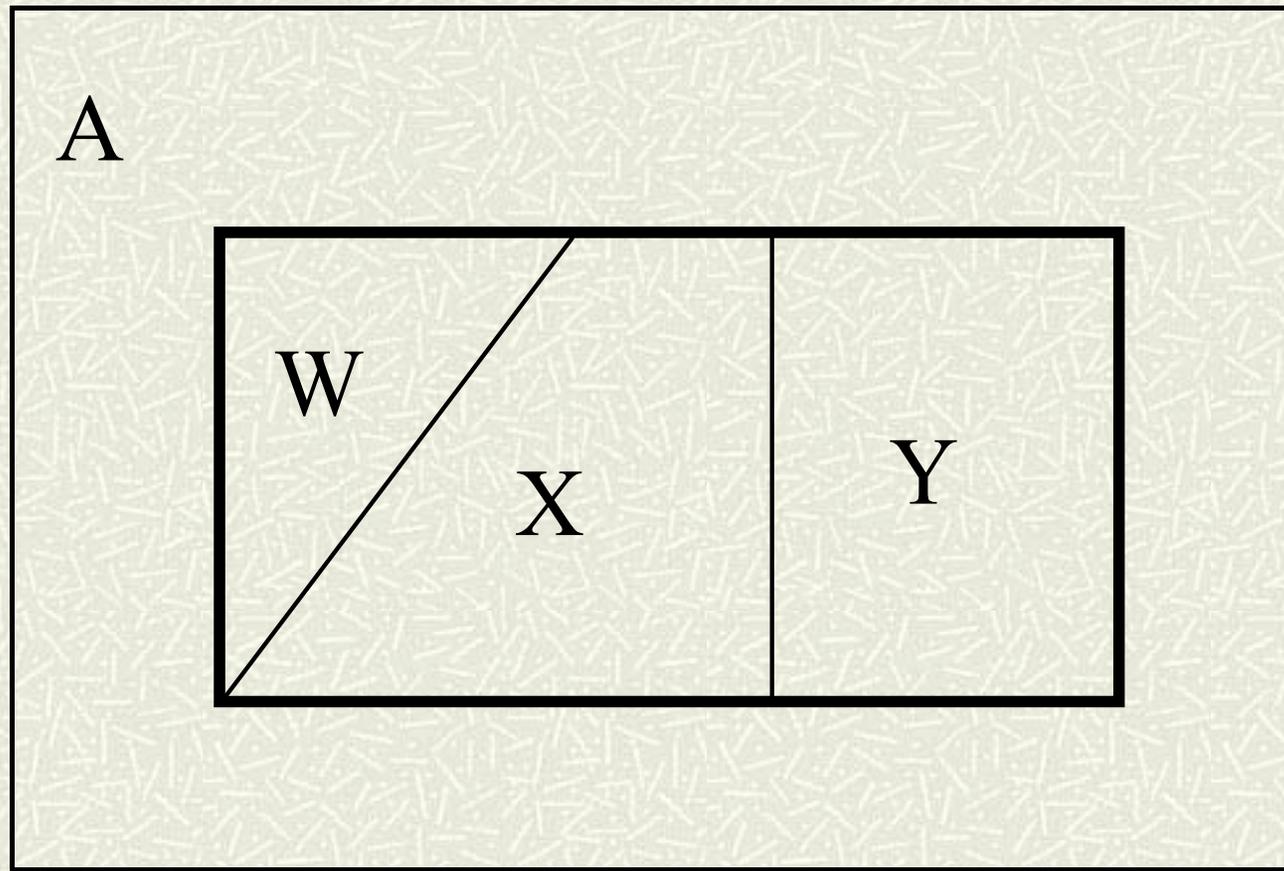
- # *Definition.* We say that a group of sets is *exhaustive* of another set if their union is equal to that set. For example, if  $A \cup B = C$  we say that A and B are exhaustive with respect to C.
  - # *Definition.* We say that two sets A and B are *mutually exclusive* if  $A \cap B = \emptyset$ , that is, the sets have no elements in common.
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# Set Partition

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- # *Definition.* We say that a group of sets *partitions* another set if they are mutually exclusive and exhaustive with respect to that set. When we “partition a set,” we break it down into mutually exclusive and exhaustive regions, i.e., regions with no overlap. The Venn diagram below should help you get the picture. In this diagram, the set A (the rectangle) is partitioned into sets W, X, and Y.
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# Set Partition



# Some Test Questions

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$$A \cup \emptyset = ?$$

# Some Test Questions

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$$A \cup \bar{A} = ?$$

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# Some Test Questions

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$$A \cap \emptyset = ?$$

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# Some Test Questions

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$A - \bar{A} = ?$

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# Some Test Questions

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$$A \cap \bar{A} = ?$$

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# Some Test Questions

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$$A \cup \Omega = ?$$

# Some Test Questions

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$A \cap \Omega = ?$

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## Some Test Questions

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If  $A \subset B$  then

$$A \cap B = ?$$

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## Some Test Questions

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If  $A \subset B$  then

$$A \cup B = ?$$

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