Name:

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Final

Some definitions: A standard TM has one tape (unbounded to the right), is deterministic, and accepts by halting in an accepting state. A TM accepts a language by halting if it halts for every string in the language and if it loops forever for strings not in the language.

You may use any symbol as markers on TM tapes.

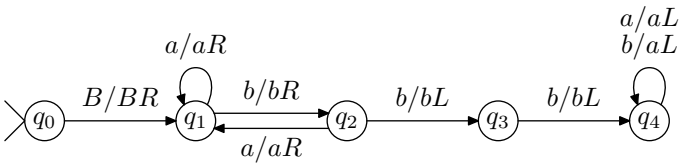
In general: if in doubt, explain what convention you are using.

1. (*8 points*) Give the first four elements in the lexicographical ordering of all strings in $\{a, b\}^*$.

Solution: λ, a, b, aa ■

2. (*15 points*) Construct a standard TM which finds the first occurrence of the string bb and then changes every nonblank symbol preceding the bb to a, leaving the bb unchanged. Explain the design of your machine.

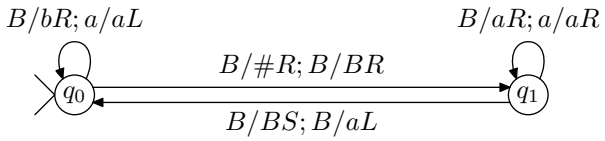
Solution: ■



The TM moves right until it reads two successive b's. Then it moves left, and after skipping the bb it just read, the TM changes all nonblank symbol to a. After all, the inital blank is read and the machine halts.

3. (*15 points*) Design a two-tape TM (or any type you like) which enumerates the language $L = \{a^i b^{i+1} \mid i \geq 0\}$.

Solution:



The TM has two tapes, which are initially empty. The first tape is an output tape, on which the TM enumerates the language as $B\#b\#abb\#aabbb\#\cdots$. The second tape is a work tape, on which the TM writes down i number of a's in Iteration i.

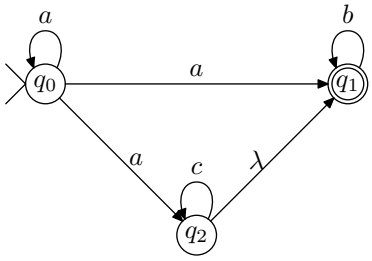
The transition from q_0 to q_1, i.e. $B/\#R; B/BR$, writes down a delimiter $\#$ on the first tape. The self-transition on q_1 copies all a's on the second tape to the first one. Then the machine goes from q_1 to q_0 with $B/BS; B/aL$ and writes down an additional a at the end of the second tape. By the transition $B/bR; a/aL$ at the state q_0, the machine writes down the same number of b's on the first tape as the number of a's on the second.

■

4. (*7 points*) Describe a problem which cannot be solved using a TM.

Solution: The halting problem is an unsolvable problem by TM. the input to the halting problem is a Turing machine and its input. The goal is to determine whether or not that Turing machine will ever reach the halt state. ■

5. (*15 points*) The following figure shows an NFA. Find an equivalent DFA.

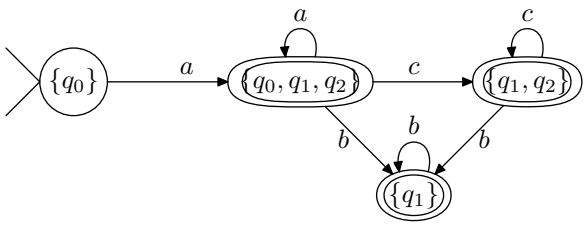


Solution:

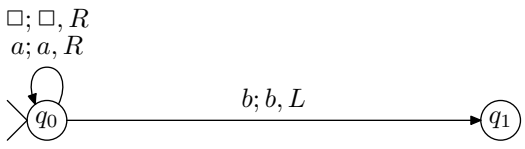
The NFA accepts the language $a^+c^*b^*$. The construction of an equivalent DFA is traced in the following table.

State	symbol	NFA Transitions	Next state
$\{q_0\}$	a	$\delta(q_0, a) = \{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$
$\{q_0, q_1, q_2\}$	a	$\delta(q_0, a) = \{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$
		$\delta(q_2, a) = \{q_1\}$	
$\{q_0, q_1, q_2\}$	b	$\delta(q_1, b) = \{q_1\}$	$\{q_1\}$
$\{q_0, q_1, q_2\}$	c	$\delta(q_2, c) = \{q_1, q_2\}$	$\{q_1, q_2\}$
$\{q_1\}$	b	$\delta(q_1, b) = \{q_1\}$	$\{q_1\}$
$\{q_1, q_2\}$	b	$\delta(q_1, b) = \{q_1\}$	$\{q_1\}$
$\{q_1, q_2\}$	c	$\delta(q_2, c) = \{q_1, q_2\}$	$\{q_1, q_2\}$

The equivalent DFA is illustrated in the following figure. ■



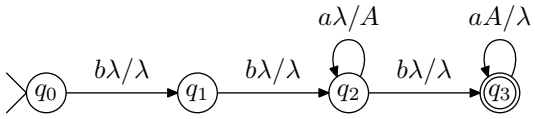
6. (*10 points*) Does the TM of the following figure accept a language? If no, give a reason. If yes, what is the language?



Solution: Yes. The accepted language is $a^*b(a \cup b)^*$. ■

7. (*15 points*) Find a PDA accepting the language $L = \{w \mid w = bba^ibba^i; i \geq 0\}$. Explain the design of the PDA.

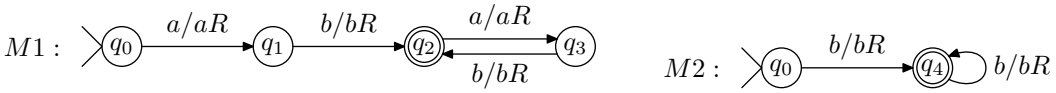
Solution:



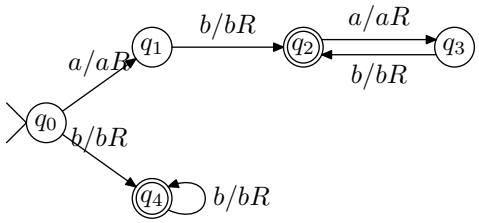
The PDA reads two successive b's at the beginning, and goes to state q_1. At q_2, it pushes the same number of A's into the stack as the number of a's it reads. Once the machine reads a b, it goes to the final state q_3, and pop up an A whenever it reads an a. ■

8. (*15 points*) Design a nondeterministic TM for the language $(ab)^+ \cup b^+$. Explain your design.

Solution: The following two machines M_1 and M_2 accepts the language $(ab)^+$ and b^+ respectively.



By merging the initial states of M_1 and M_2, we obtains the TM that accepts the language $(ab)^+ \cup b^+$.



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