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Midterm

Note: B-spline knots are given using Mathematica notation.

1. Consider the knot sequence 0,0,0,0,1,2,3,4,4,4,4. Sketch a convex control polygon such that a cubic B-spline curve is defined. Sketch the curve.

2. Illustrate the convex hull property for degree a four Bézier curve. The curve should have a nonconvex control polygon.

3. Sketch the degree 4 Lagrange polynomials L_0^4 and L_2^4 over $t_0, \ldots, t_4 = 0, 1, 3, 5, 6$.

4. Consider the knot sequence 0,0,0,0,2,4,4,4,4. Given data points $[0, 0], [1, 0], [2, 1], [2, 2], [3, 2], [4, 2]$ with parameter values 0,1,2,3,3.5,4, what is the Vandermonde matrix for the corresponding cubic least squares approximation problem? (Do not evaluate B-splines – just write down B-spline and parameter value at which to evaluate).

5. Sketch a cubic Bézier curve + polygon with zero second derivative at $t = 0$.

6. What is the dimension of the spline space of piecewise linear polynomials over 0,0,3,3.5,4,4?

7. Consider the Bézier control polygon [0,0], [0,1], [1,1], [1,0]. Sketch its first derivative curve with corresponding control polygon. Also sketch in the first derivative vector at $t = \frac{1}{2}$, both on the cubic curve and the first derivative curve.

8. Find a 3D cubic Bézier curve (nonplanar) whose projection into the x, y-plane has an inflection point (i.e., it is S-shaped). Sketch and print the control point coordinates.