B-spline Blossoms

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1 De Boor and Greville

As we insert the knot (parameter value) u more times, we obtain a hierarchy of Greville abscissae ξ_i^r, given by

$$\xi_i^r = \frac{1}{n} \sum_{j=i}^{i+n-r-1} u_j \quad + \quad \frac{r}{n} u; \quad i = I-n+r+1, \ldots, I+1. \tag{1}$$

For $r = n$, this reduces to $\xi_{I+1}^n = u$.

In the de Boor algorithm, every d_i^r is computed by linear interpolation of d_{i-1}^{r-1} and d_i^{r-1} at parameter value ξ_i^r:

$$d_i^r = \frac{\xi_i^{r-1} - \xi_i^r}{\xi_i^{r-1} - \xi_{i-1}^{r-1}} d_{i-1}^{r-1} + \frac{\xi_i^r - \xi_{i-1}^{r-1}}{\xi_i^{r-1} - \xi_{i-1}^{r-1}} d_i^{r-1}. \tag{2}$$

Thus d_i^r is obtained by linear interpolation over the interval $[\xi_{i-1}^{r-1}, \xi_i^{r-1}]$. After replacing all intermediate Greville abscissae by their definition (1), the "usual" form of the de Boor algorithm is retrieved.

2 Derivatives

The last de Boor step,

$$d_{I+1}^n = \frac{u_{I+1} - u}{u_{I+1} - u_I} d_I^{n-1} + \frac{u - u_I}{u_{I+1} - u_I} d_{I+1}^{n-1}$$

is a de Casteljau step with respect to the interval $[\xi_I^{n-1}, \xi_{I+1}^{n-1}] = [u_I, u_{I+1}]$. Hence the derivative of the curve is given by

$$\dot{d}(u) = \frac{n}{u_{I+1} - u_I}[d_{I+1}^{n-1} - d_I^{n-1}].$$

Note that this is a difference step with respect to the 1D vector $\vec{1}$. Using blossoms, this becomes:

$$\dot{d}(u) = n d_I[u^{<n-1>}, \vec{1}].$$

Of course we may carry out the difference step at any level of the blossom scheme. In particular, we may utilize it at the first level. This gives the derivative formula on p. 168 of the book.