Notes on Geometric Continuity

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1 Memke's Theorem

With the notation of the corrected Fig. 12.2, and setting $l_- = \|\mathbf{b}_3 - \mathbf{b}_2\|$, $l_+ = \|\mathbf{c}_1 - \mathbf{b}_3\|$, we have

$$\frac{\kappa_-}{\kappa_+} = \frac{\mathcal{A}_-}{\mathcal{A}_+}\frac{l_+^3}{l_-^3} = \frac{\mathcal{A}_-}{\mathcal{A}_+}\frac{\mathcal{B}_+}{\mathcal{B}_-}\frac{l_-}{l_+}\frac{l_+^3}{l_-^3}.$$

Thus

$$\frac{\kappa_-}{\kappa_+} = \frac{r_- r_+}{r^2}.$$

This is known as *Memke's theorem* and states that the ratio of left and right curvatures on a ny point of a curve is invariant under affine maps. This follows since we only use affinely invariant ratios in the formulation of $\kappa_-/kapp_+$.

2 Quick ν-splines

If a C^1 curve $\mathbf{x}$ is curvature continuous at a point $\mathbf{x}(u)$, then we must have

$$\|\ddot{\mathbf{x}}_- \wedge \dot{\mathbf{x}}\| = \|\ddot{\mathbf{x}}_+ \wedge \dot{\mathbf{x}}\|.$$

Thus the three involved vectors must satisfy a relationship

$$\ddot{\mathbf{x}}_+ - \ddot{\mathbf{x}}_- = \nu\dot{\mathbf{x}}.$$

The constant ν depends on the parametrization of the curve; if we change u to $k\dot{u}$, we will have to replace ν by $k\dot{\nu}$.