Integrals

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The derivative of a polynomial $b(t)$ in Bernstein form is given by

$$\dot{\mathbf{x}}(t) = n \sum_{i=0}^{n-1} \Delta b_i B_i^{n-1}(t).$$

In order to find the indefinite integral, or antiderivative, we must invert this process. So if a polynomial in Bernstein form is given by its control ordinates

$$b_0, \ldots, b_n, \tag{1}$$

then the set of control ordinates

$$\frac{1}{n+1}[c, c+b_0, c+b_0+b_1, \ldots, c+\sum_{i=0}^{n} b_i]$$

defines a polynomial $B(t)$ whose derivative is exactly the one defined by the control ordinates (1). The scalar c is the usual constant encountered in indefinite integration.
Since

$$\int_0^1 b(t)\mathrm{d}t = B(1) - B(0), \tag{2}$$

we have immediately

$$\int_0^1 b(t)\mathrm{d}t = \frac{1}{n+1} \sum_{i=0}^{n} b_i. \tag{3}$$