G. Farin

Programming Assignment # 3

due: 4-19

Define a set of noisy data points $\mathbf{p}_i$ by setting

$$\mathbf{p}_i = \begin{bmatrix} (2+\cos(3t_i/2))\cos(t_i) + \sin(100t_i)/10 \\ (2+\cos(3t_i/2))\sin(t_i) + \cos(200t_i)/10 \\ \sin(3t_i/2) \end{bmatrix}$$

for $0 \le t_i \le 4\pi$.

Without the noise terms, this is called a *trefoil knot.*

Generate 1,000 points (or so) on the curve and then find a least square B-spline approximation. Use cubic and quartic B-spline curves and experiment with the number of intervals so as to achieve a good fit. Also experiment with data having gaps.

Also experiment with shape equations (covered next class).

Hand in various (postscript) plots or demonstrate in class.

Here are the B-spline recursions:

de Boor algorithm: Given $u_I \le u < u_{I+1}$, renumber the relevant control points $\mathbf{d}_{I-n+1}, \ldots, \mathbf{d}_{I+1}$ as $\mathbf{d}_0, \ldots \mathbf{d}_n$ and then set

$$\mathbf{d}_i^k(u) = (1-\alpha_i^k)\mathbf{d}_i^{k-1}(u) + \alpha_i^k \mathbf{d}_{i+1}^{k-1}(u)$$

with

$$\alpha_i^k = \frac{u - u_{I+i+1}}{u_{I-n+k+i} - u_{I+i+1}}$$

for $k = r+1, \ldots, n,$ and $i = 0, \ldots, n-k$. Here, r denotes the multiplicity of u. (Normally, u is not already in the knot sequence; then, $r = 0$.)

Mansfield, de Boor, Cox recursion:

$$N_l^n(u) = \frac{u - u_{l-1}}{u_{l+n-1} - u_{l-1}} N_l^{n-1}(u) + \frac{u_{l+n} - u}{u_{l+n} - u_l} N_{l+1}^{n-1}(u).$$

Use this recursion until you reach $n = 1$ and then switch to the explicit piecewise linear definition of the linear basis functions.